Probability

Probability: is the study of random or nondeterministic experiments

Historically, probability theory began with study of games of chance, such as roulette (لعبة قمار) and cards

The probability "p" of an event "A" was defined as follows: If A can occur in "s" ways out of total of n=

$$\boxed{p = P(A) = \frac{s}{n}}$$

"n = equally likely" عدد مرات المحاولات

"s = number of success" احتمالية النجاح

EX in tossing (قذف) a die (نرد) an even number can occur in 3 ways out of 6 "equally likely" ways

$$p = \frac{3}{6} = \frac{1}{2}$$

SAMPLE SPACE AND EVENTS

* The set "S" of all possible outcomes of some given experiment is "sample space"

عدم احد الحدث

* "$\emptyset$" is the empty set or "impossible event"

"event A" is a set of outcomes

Since an event is a set, we combine events to form new event using various set operations

(i) $A \cup B$ is event (حدث), that occurs iff "A" occurs or B occurs, or Both

(ii) $A \cap B$ is event that occurs iff A occurs and B occurs

(iii) A^c, the complement of A also written $\bar{A}$, is the event that occurs iff A does not occur

احتماليه عدم حدوث مثلا A
الحدث يحدث فقالة

mutually exclusive (التبادل المنفي)

If two events A and B are called mutually exclusive if they are disjoint

$A \cap B = \phi$ is mutually exclusive iff they cannot occur simultanously

قذف نرد راقب

Ex Experiment: Toss a die and observe the number that appears on Top. Then the sample space Consists of the six possible numbers

Sol

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let "A" be the event that an even number occurs

"B" that and odd number occurs

"C" that a prime number occurs

$A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$ $C = \{2, 3, 5\}$

في الاتحاد المتشابه يؤخذ مرة واحدة

Then

$A \cup C = \{2, 3, 4, 5, 6\}$ is the event that an even or a prime number occurs

$B \cap C = \{3, 5\}$ is the event that an odd prime number occurs

$C^c = \{1, 4, 6\}$ is the event that a prime number does not occur

Not that A and B are mutually exclusive $A \cap B = \phi$

$B \cap C$ ← عدد أولي وفردي

$B \cup C$ ← إما عدد فردي أو عدد أولي أو كلاهما (فردي وأولي)

EX Let A and B be events. Find an expression and exhibit the Venn diagram for the event that

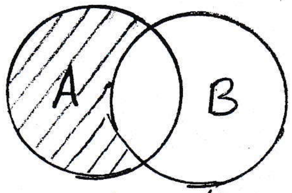
i) A but not B occurs i.e only A occurs

ii) either A or B, but not both occurs i.e exactly one of the two events occurs

Sol

(i) Shade the area of A out side of B as in fig (a) below

$A \cap B^c$



A but not B occurs

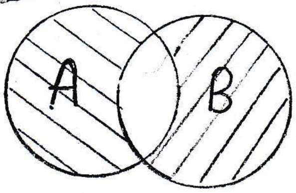
Fig (a)

(ii) since A or B not both occurs

A but not B $\quad A \cap B^c$

B but not A $\quad B \cap A^c$

Thus $(A \cap B^c) \cup (B \cap A^c)$



Either A or B, but not both occurs

Fig (b)

عملة نقود

EX Let a coin and a die be tossed, let the sample space consist of twelve elements

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6$

موضح

i) Express explicitly the following events:

$A = \{$heads and an even number apears$\}$

$B = \{$a prime number appears$\}$

$C = \{$tails and an odd number appears$\}$

مبين

ii) Express explicitly the event that: (a) ~~A or B occurs~~ (b) "B" and "C" occurs (c) only "B" occurs

هنا يعني أي شيء تقع A أو يحدث

iii) which of the events A, B, and C are mutually exclus

أي منهم يمكن

Sol

i) To obtain A, choose those elements of S

$A = \{H2, H4, H6\}$

$B = \{H2, H3, H5, T2, T3, T5\}$

$C = \{T1, T3, T5\}$

ii) (a) A or B $= A \cup B = \{H_2, H_4, H_6; H3, H5, T2, T3, T5\}$

(b) B and C $= B \cap C = \{T3, T5\}$

(c) $B \cap A^c \cap C^c = \{H3, H5, \cancel{T2}\}$

iii) A and C are mutually exclusive since $A \cap C = \phi$

Finite Probability Spaces

Let S be a finite sample space $S = \{a_1, a_2, \dots, a_n\}$.
A finite probability space obtained by assigning to each point $a_i \in S$ a real number p_i call the probability of a_i

i) each p_i is non-negative $p_i \geq 0$

ii) the sum of the p_i is one $p_1 + p_2 + \dots + p_n = 1$

لو جمعنا احتماليه حدوث حدث معين من مجموعة احداث = 1

ولاهيه وهو امثال دلعه — امنا حرول HI ٢ — ولدللي ١ — كلس c — 8

EX Let three coins be tossed and number of heads observed then the sample space is $S=\{0,1,2,3\}$ we obtain a probability space by the following assignment

sol

$$P(0)=\frac{1}{8},\ P(1)=\frac{3}{8},\ P(2)=\frac{3}{8} \text{ and } P(3)=\frac{1}{8}$$

Let "A" be the event that at Least one head appears

$A=\{1,2,3\}$ المجموعة التي تحتمل ان يظهر بها ال Head

"B" the event that all heads or all tails appear

$B=\{0,3\}$ Ta

Then by definition

$$P(A)=P(1)+P(2)+P(3)=\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=\frac{7}{8}$$

$$P(B)=P(0)+P(3)=\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$$

T T T H H H

Tails	head
3	0
2	1
1	2
0	3

$P(0)=1 \to$ all Taill

$P(1) \to$ First, second, Third

$P(2) \to$ first + second, Second + Third, first + third

$P(3) \to$ all head

EX Let a die be weighted so that the probability of number appearing when the die is tossed is proportional to the given number (e.g. 6 has twice the probability of appearing a 3). Let $A=\{\text{even number}\}$, $B=\{\text{prime number}\}$, $C=\{\text{odd n}$

(i) Describe the probability space i.e find the probability of each sample point وزن الوجه = الاحتمالية

(ii) Find $P(A)$, $P(B)$ and $P(C)$.

(iii) Find the probability that (a) an even or prime number occurs (b) an odd prime number occurs (c) A but not B occurs

اضمانه صلوا على النبي

Sol (i) Let $P(1) = P$ then $P(2) = 2P$, $P(3) = 3P$, $P(4) = 4P$

$P(5) = 5P$, $P(6) = 6P$

but sum of probability $= 1$

$\therefore P + 2P + 3P + 4P + 5P + 6P = 1 \Rightarrow P = \frac{1}{21}$ then ...

$P(1) = \frac{1}{21}$, $P(2) = \frac{2}{21}$, $P(3) = \frac{1}{7}$, $P(4) = \frac{4}{21}$, $P(5) = \frac{5}{21}$, $P(6) = \frac{6}{21}$

(ii) عدد زوجي عدد أولي عدد فردي

$P(A) = P(2,4,6) = \frac{4}{7}$, $P(B) = P(2,3,5) = \frac{10}{21}$, $P(C)\{1,3,5\} = \frac{3}{7}$

(iii) (a) The event that an even or prime number occurs is. Thus $P(A \cup B) = \{2,4,6,3,5\} = 1 - P(1) = \frac{20}{21}$

(b) The event that an odd prime number occurs is $B \cap C = \{3,5\}$. Thus $P(B \cap C) = P\{3,5\} = \frac{8}{21}$

(c) The event that A but not B occurs $A \cap B^c = \{4,6\}$

Hence $P(A \cap B^c) = P\{4,6\} = \frac{10}{21}$

Equiprobably spaces

If a finite probability space S, where each sample point has the same probability, will be called "equiprobable space"

- * If S contains n points then the probability of each point $\frac{1}{n}$
- * If an event A contains r points then its probability is

$$r \cdot \frac{1}{n} = \frac{r}{n} \quad \text{or} \quad P(A) = \frac{\text{number of element in } A}{\text{number of element in } S} = \frac{\text{number of ways that the event } A}{\text{number of ways that the sample space } S \text{ can occur}}$$

Compliments (6)

EX Let 2 items be chosen at random from a lot containing 12 items of which 4 are defective. Let

A = {both items are defective} and B = {both items are non defective}

Find P(A) and P(B)

$$x^n \qquad \text{probability} = \frac{n!}{(n-x)! \ast \cancel{x}}$$

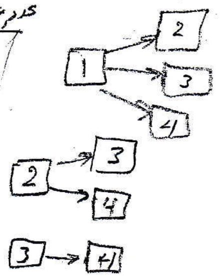
Sol S can occur $\binom{12}{2} = 66$ ways the number of ways that 2 items can be chosen from (12) items

1 2 3 4 5 6 7 8 9 10 11 12

رقم الصوره	عدد الاحتمالات	رقم الصوره	عدد الاحتمالات	رقم الصورة	عدد الاحتمالات		
1 →	(11)	4 →	(8)	7 →	(5)	10 →	(2)
2 →	(10)	5 →	(7)	8 →	(4)	11 →	(1)
3 →	(9)	6 →	(6)	9 →	(3)		
	30 +		21 +		12		3 = 66

"A" can occur in $\binom{4}{2} = 6$ ways the number of ways that 2 defective items can be chosen from 4 defective items

No. pick رقم الصوره	No. way عدد الاحتمالات
1	3
2	2
3	1
	6

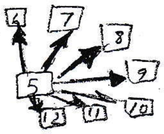


Let

$\underbrace{1\ 2\ 3\ 4}_{\text{defective}}\ \underbrace{5\ 6\ 7\ 8\ 9\ 10\ 11\ 12}_{\text{not defective}}$

"B" can occur in $\binom{8}{2} = 28$ ways, the number of ways that 2 non-defective items can be chosen from 8 non-defective items

No of picher	No. way
5	7
6	6
7	5
8	4
9	3
10	2
11	1
	28



$\therefore P(A) = \frac{6}{66} = \frac{1}{11}$ and $P(B) = \frac{28}{66} = \frac{14}{33}$

EX Two Cards are drawn at random from ordinary deck of 52 Cards. Find the probability p that

i) both are spades (ii) one is a spade and one is a heart

There are $\binom{52}{2} = 1326$ ways to draw 2 cards from 52 cards

sol

i) There are $\binom{13}{2} = 78$ ways to draw 2 spades from 13 spades

hence $p = \dfrac{\text{number of ways 2 spades can be drawn}}{\text{number of ways 2 Cards can be drawn}} = \dfrac{78}{1326} = \dfrac{3}{51}$

ii) since there 13 spades and 13 hearts there are $13 * 13 = 169$ ways to draw a spade and a heart

hence $p = \dfrac{169}{1326} = \dfrac{13}{102}$

EX six married couples are standing in a room

(i) If 2 people are chosen at random, find the probability p that (a) they are married (b) one is male and one is female

(ii) If 4 people are chosen at random, find the probability that (a) 2 married couples are chosen (b) no married couple is among the 4 (c) exactly one married couple is among the 4.

(iii) If the 12 people are divided into six pairs, find the probability p that (a) each pair is married (b) each pair contains a male and a female.

$\frac{12!}{(12-4)!\times 4}$

(i) There are $\binom{12}{2} = 66$ ways to choose 2 people from the 12 people

(a) There are 6 married couples; hence $p = \frac{6}{66} = \frac{1}{11}$

(b) There are 6 way to choose a male and 6 ways to choos a female; hence $p = \frac{6 * 6}{66} = \frac{6}{11}$

(ii) There are $\binom{12}{4} = 495$ ways to choose 4 people from 12 peop

(a) There are $\binom{6}{2} = 15$ ways to choose 2 couples from the 6 couples hence $p = \frac{15}{495} = \frac{1}{33}$

(b) The 4 persons come from 4 different couples. There are $\binom{6}{4} = 15$ ways to choose 4 couples from the 6 couples, and there are 2 ways to choos one person from each couple. Hence

$p = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 15}{495} = \frac{16}{33}$

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(c) This event is mutually disjoint from the precedir two events (which are also mutually disjoint) and at least one of these events must occurs Hence $p + \frac{1}{33} + \frac{16}{33} = 1$ or $p = \frac{16}{33}$

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(393)

THEOREMS ON FINITE PROBABILTY SPACES

Theorems :-(1) :-

The probability function P defined on class of all events in a finite probability space satisfies the following axioms

[P1] For every event A $0 \leq P(A) \leq 1$

[P_2] $P(S) = 1$

[P_3] If events A and B are mutally exclusive then $P(A \cup B) = P(A) + P(B)$

Using mathematical induction [P_3] can be generalize as follows

Theorem 2 If

If $A_1, A_2 \ldots A_r$ are pairwise mutually exclusive event then

$$P(A_1 \cup A_2 \cup \ldots \cup A_r) = P(A_1) + P(A_2) + \ldots + P(A_r)$$

Theorem 3

If ϕ is empty set, and A and B are arbitrary event then

i) $P(\phi) = 0$

ii) $P(A^c) = 1 - P(A)$

iii) $P(A \setminus B) = P(A) - P(A \cap B)$ ie $P(A \cap B^c) = P(A) - P(A \cap B)$

iv) $A \subset B$ implies $P(A) \leq P(B)$